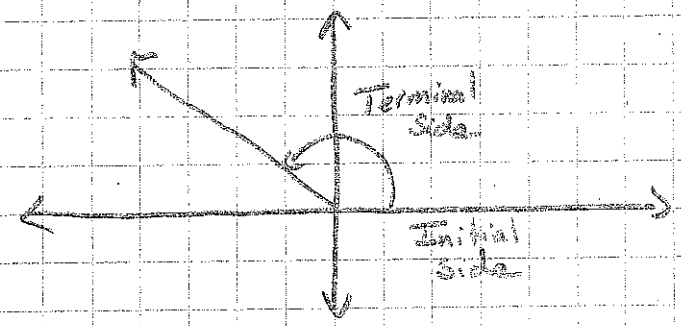
Unit 4 Trig Functions

4.1 Angles and Degree Measure

Objective: You will "review" your first 3 years of Basic trig Info.

Angle Measure



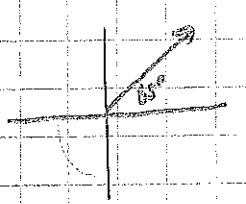
Quadrantal Angles: $\pm 90°, \pm 180°, \pm 270°, \pm 360°$

Coterminal Angles: If α is the degree measure of an angle, then all angles measuring $\alpha \pm 360K°$, where K is an integer, are coterminal with α (share the same terminal side).

Examples:

1) Identify all the angles that are coterminal with 85°.

Ans: $85° \pm 360K°$



- Stole 1 positive + 1 negative coterminal ∠.

Positive: $85° + 360(1) = \boxed{445°}$

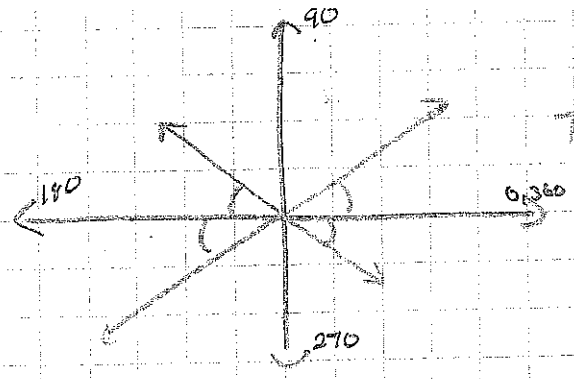
Negative: $85 - 360(3) = \boxed{-995°}$

2) Find the smallest positive coterminal angle to 775°.

$775 - 360 = 415 - 360 = \boxed{55°}$

3) Find the smallest positive coterminal angle to $-685°$.

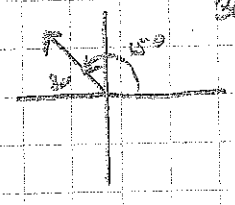
$\boxed{\quad}$



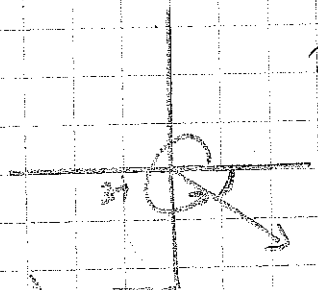
Think "Four-ths"

Find the reference angle of:

1) 150°

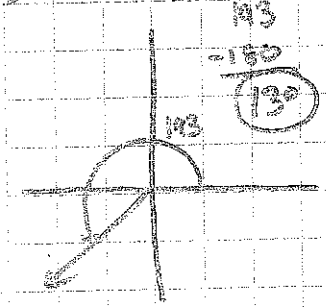


2) 317°

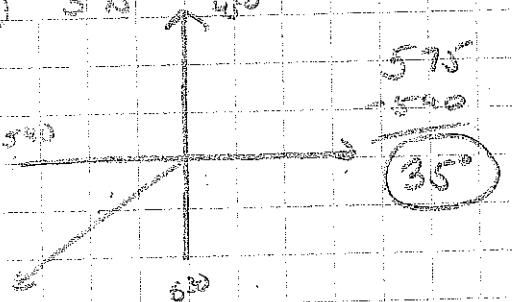


360 - 317 = 43°

3) 193°

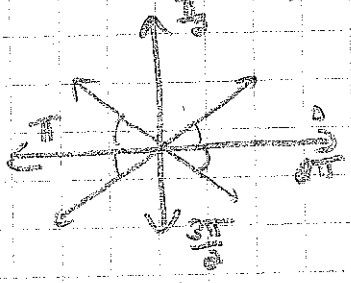


4) 575°



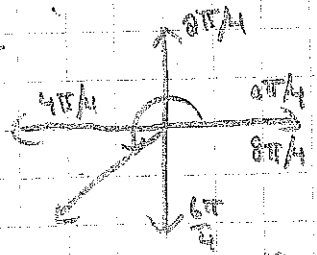
575
-540
35°

Reference Angles



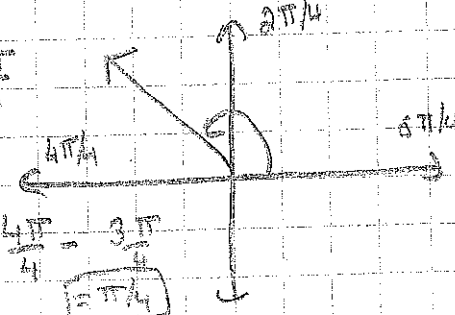
Find the reference angle.

1) $\frac{5\pi}{4}$



$\frac{5\pi}{4} - \frac{4\pi}{4} =$ $\frac{\pi}{4}$

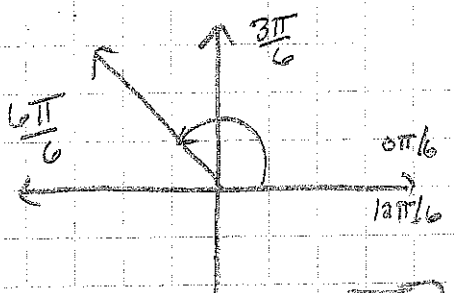
2) $\frac{3\pi}{4}$



$\frac{4\pi}{4} - \frac{3\pi}{4}$

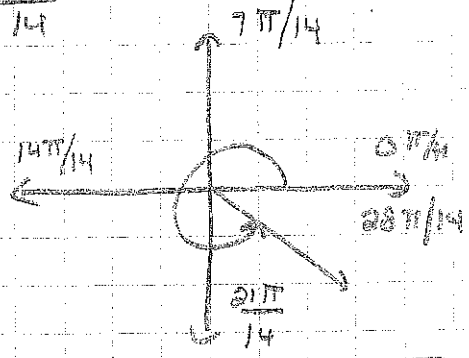
$\frac{\pi}{4}$

3) $\frac{5\pi}{6}$



$\frac{6\pi}{6} - \frac{5\pi}{6} = \boxed{\pi/6}$

4) $\frac{25\pi}{14}$



$\frac{28\pi}{14} - \frac{25\pi}{14} = \boxed{\frac{3\pi}{14}}$

HW: Find the smallest positive coterminal angle

1) $-190°$ 2) 500 3) $-624°$

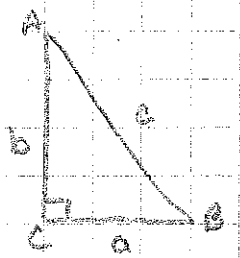
4) Find the reference angle of

a) 217 b) $157°$

c) $\frac{7\pi}{4}$ d) $\frac{11\pi}{6}$

Unit 4 Day #2 – Trig Ratios in Right Triangles

Objective – You will find the values of trig ratios for acute angles of right Δ's.



Legs: a, b

hypotenuse: c

with respect to angle B:

a is adjacent
b is opposite
c is hypotenuse

Trig Ratios: $\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

Reciprocal Trig Ratios

$\csc\theta = \frac{1}{\sin\theta}$ $\frac{H}{O}$ $\sec\theta = \frac{1}{\cos\theta}$ $\frac{H}{A}$ $\cot\theta = \frac{1}{\tan\theta}$ $\frac{A}{O}$

Examples:

1) If $\tan\theta = \frac{1}{3}$, find $\cot\theta$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{3}} = \frac{1}{1}\cdot\frac{3}{1} = \boxed{3}$

2) If $\cot\theta = .75$, find $\tan\theta$

$\tan\theta = \boxed{\frac{4}{3}}$

3) Find:

$q^2 + 7^2 = (2\sqrt{22})^2$

$q^2 + 49 = 88$

$\sqrt{q^2} = \sqrt{39}$

$q = \sqrt{39}$

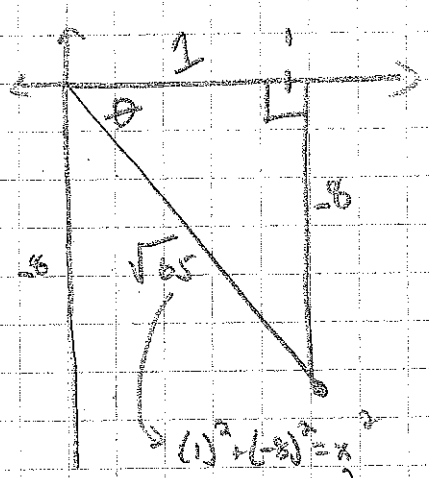
a) $\sin R = \frac{O}{H} = \frac{\sqrt{7}}{2\sqrt{22}}$

b) $\cos R = \frac{A}{H} = \frac{\sqrt{39}}{2\sqrt{22}}$

d) $\csc R = \frac{2\sqrt{22}}{\sqrt{7}}$

e) $\sec R = \frac{2\sqrt{22}}{\sqrt{39}}$

4) Angle θ goes through the point $(1, -8)$
Evaluate the 6 trig functions for θ



$\sin\theta = \frac{o}{h} = \frac{-8}{\sqrt{65}} \cdot \frac{\sqrt{65}}{\sqrt{65}} = \boxed{\frac{-8\sqrt{65}}{65}}$

$\cos\theta = \frac{a}{h} = \frac{1}{\sqrt{65}} \cdot \frac{\sqrt{65}}{\sqrt{65}} = \boxed{\frac{\sqrt{65}}{65}}$

$\tan\theta = \frac{o}{a} = \frac{-8}{1} = \boxed{-8}$

$\csc\theta = \frac{h}{o} = \boxed{\frac{\sqrt{65}}{-8}}$

$\sec\theta = \frac{h}{a} = \frac{\sqrt{65}}{1} = \boxed{\sqrt{65}}$

$\cot\theta = \frac{a}{o} = \frac{1}{-8} = \boxed{\frac{-1}{8}}$

Radians and Degrees

Converting from radians to Degrees:

1) multiply by $\frac{180}{\pi}$

2) Simplify

ex: $\frac{4\pi}{5}$

$\frac{4\pi}{5} \cdot \frac{180}{\pi} = \boxed{144^\circ}$

ex: $\frac{13\pi}{10}$

$\frac{13\pi}{10} \cdot \frac{180}{\pi} = \boxed{234^\circ}$

Converting from degrees to radians:

1) Multiply by $\frac{\pi}{180}$

2) Simplify

HW: See Slide

ex: 225°

$225^\circ \cdot \frac{\pi}{180}$

$\boxed{\frac{5\pi}{4}}$

ex: 140°

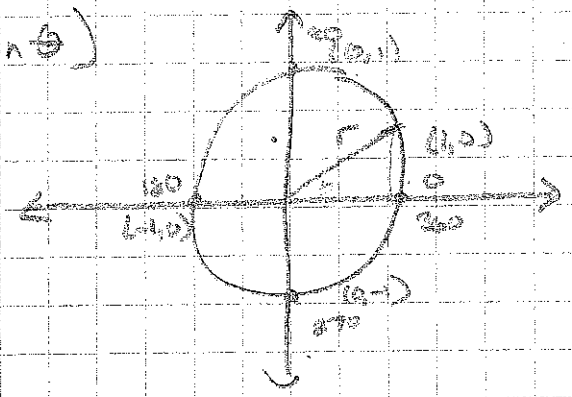
$140 \cdot \frac{\pi}{180} = \boxed{\frac{7\pi}{9}}$

Unit 4 Day #3 - Trig Functions at the Unit Circle

Objective: You will use the unit circle and special right triangles to find trig. values.

$(x, y) = (\cos\theta, \sin\theta)$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$



Evaluate:

1) $\sin 90° =$ 1

2) $\tan 360 = \frac{0}{1} =$ 0

3) $\cot(-180°) = \frac{\cos\theta}{\sin\theta} = \frac{-1}{0} =$ (undefined)

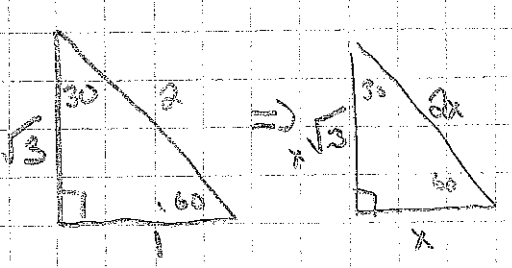
4) $\csc 270° = \frac{1}{\sin 270} = \frac{1}{-1} =$ -1

5) $\cos(-270°) =$ 0

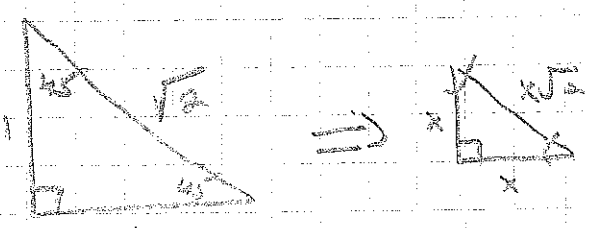
6) $\sec 180° = \frac{1}{\cos 180} = \frac{1}{-1}$ = -1

Special Right Triangles

30-60-90



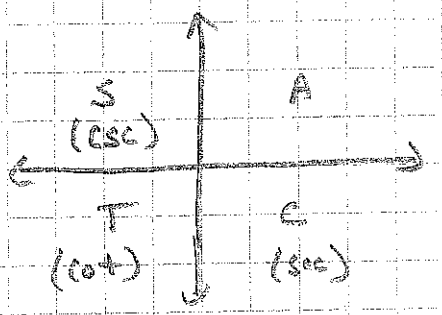
45-45-90



Finding Exact Values of Trig Functions

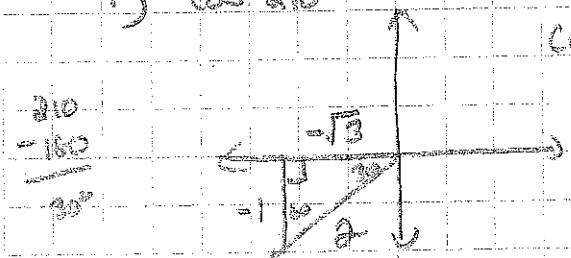
HW See Slide

1) Find the reference angle
2) Use a special right triangle to find the exact value.
3) Determine if the answer is + or − based on the original quadrant.



Find the exact value:

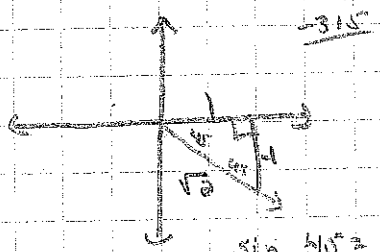
1) $\cos 210°$



$\cos 30° = \frac{\sqrt{3}}{2}$

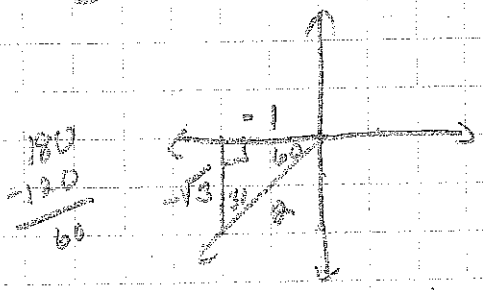
$\cos 210° = -\frac{\sqrt{3}}{2}$

2) $\sin 315$



$\sin 45° = \frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

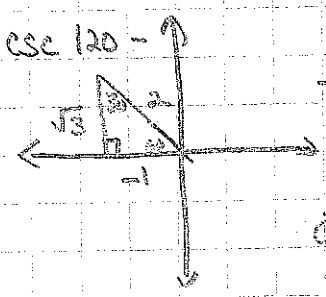
3) $\tan(-120)$



$\tan 60° = \frac{\sqrt{3}}{1}$

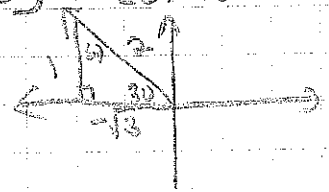
$= -\frac{\sqrt{3}}{-1} = \sqrt{3}$

4) $\csc 120 =$



$\csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

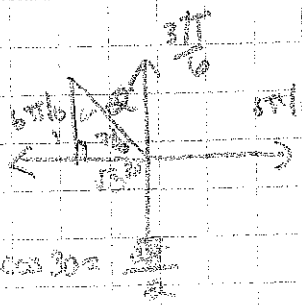
5) $\cot 150°$



$\cot 30° = \frac{\sqrt{3}}{1}$

$= -\sqrt{3}$

6) $\cos \frac{5\pi}{6}$



$\frac{5\pi}{6} \cdot \frac{180}{\pi}$

$\cos 30° = \frac{\sqrt{3}}{2}$

Unit 4 Day #4 - Finding Missing Trig Functions

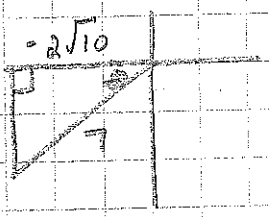
Objective - You will find all six trig values of an angle in standard position given its terminal side

Finding Missing Trig Functions

You will be given the value of one trig function and the quadrant in which the angle lies. You will have to evaluate the remaining trig. functions for that angle.

1) Draw a sketch based on the given information
2) Find the missing side of the triangle using Pyth. Theorem
3) Evaluate the remaining trig functions based on the diagram.

Ex: 1) If $\sin\theta = \frac{-3}{7}$ and θ is in Quadrant III, find the 5 missing trig functions



$a^2 + (-3)^2 = (7)^2$

$a^2 + 9 = 49$

$\sqrt{a^2} = \sqrt{40}$

$a = \sqrt{4}\sqrt{10}$

$a = 2\sqrt{10}$

$\cos\theta = \frac{A}{H} = \boxed{\frac{-2\sqrt{10}}{7}}$

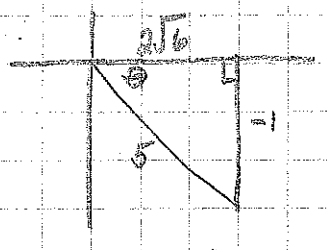
$\tan\theta = \frac{O}{A} = \frac{-3}{-2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{4(10)} = \frac{3\sqrt{10}}{40} = \boxed{\frac{3\sqrt{10}}{20}}$

$\csc\theta = \boxed{\frac{-7}{3}}$

$\sec\theta = \frac{H}{A} = \frac{-7}{-2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{-7\sqrt{10}}{4(10)} = \frac{-7\sqrt{10}}{40} = \boxed{\frac{7\sqrt{10}}{20}}$

$\cot\theta = \frac{A}{O} = \frac{-2\sqrt{10}}{-3} = \boxed{\frac{2\sqrt{10}}{3}}$

2) If $\csc\theta = -5$ and θ is in Quadrant IV, find the 5 missing trig functions.



$\csc\theta = -5$

$\sin\theta = \boxed{-\frac{1}{5}}$

$\cos\theta = \frac{A}{H} = \boxed{\frac{2\sqrt{6}}{5}}$

$(-1)^2 + a^2 = 5^2$

$a^2 + 1 = 25$

$a^2 = 24$

$a = \sqrt{24}$

$\boxed{a = 2\sqrt{6}}$

$\tan\theta = \frac{O}{A} = \frac{-1}{2\sqrt{6}} \cdot \frac{2\sqrt{6}}{2\sqrt{6}} = \frac{-2\sqrt{6}}{4(6)} = \frac{-2\sqrt{6}}{24} = \boxed{\frac{-\sqrt{6}}{12}}$

$\sec\theta = \frac{H}{A} = \frac{5}{2\sqrt{6}} \cdot \frac{2\sqrt{6}}{2\sqrt{6}} = \frac{10\sqrt{6}}{4(6)} = \frac{10\sqrt{6}}{24} = \boxed{\frac{5\sqrt{6}}{12}}$

$\cot\theta = \frac{A}{O} = \frac{2\sqrt{6}}{-1} = \boxed{-2\sqrt{6}}$